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Heaviside-Maxwellian Gravity form Local U(1) Phase/ Gauge Invariance of the Lagrangian for Massive Neutral Dirac Particle

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Abstract. Starting with Dirac Lagrangian for free massive electrically neutral Dirac particles and demanding local phase invariance, we are forced to introduce a massless vector field (A_g^μ) and to find a complete Lagrangian that generates all of Heaviside's Gravity (HG) of 1893 as well as the relativistic Maxwellian Gravity (MG) which specifies the current produced by massive electrically neutral Dirac particles. The resulting spin-1 vector gravitational theory, named here as Heaviside-Maxwellian Gravity (by establishing the equivalence of HG and MG), is shown to produce attractive force between like masses under static condition, contrary to the prevalent view on vector gravitational theory.

Keywords. Heaviside-Maxwellian Gravity, Gravitomagnetism, Attraction in Spin-1 Vector Gravity, Gravitational Waves (GWs), Speed of GWs.

1. Introduction

Many field theorists, like Gupta [1], Feynman [2], Low [3], Padmanabhan [4], Zee [5], Gasperini [6] and Straumann [7] state that gravity cannot be described by a spin-1 vector theory like Maxwell's theory of electromagnetism because a spin-1 vector gravitational theory yields repulsion between two static masses contrary to Newtonian gravitational interaction between two static masses. However, here we show that this is not true, if one considers appropriate field equations for vector gravity. We follow the usual procedure of quantum electrodynamics in flat space-time. Starting with the free Lagrangian for a massive electrically neutral Dirac particle of rest mass m_0 and applying the requirement of local phase invariance, we find an invariant Lagrangian that generates all of gravitodynamcis of Maxwellian Gravity (MG) [8,9,10] and also specifies the current produced by massive Dirac particles. Alongside, we also rediscovered little known Heaviside's Gravity (HG) [11, 12, 13, 14, 15, 16] of 1893, which is shown here to be equivalent to MG, if the fields and potentials are defined properly. The resulting theory, spin-1 Heaviside-Maxwellian Gravity (HMG) is shown to produce attractive force between like masses under static condition.

Units and Notations: Here we use SI units so that the paper can easily be understood by general readers. The flat space-time symmetric metric tensor $\eta_{\alpha\beta} = \eta^{\alpha\beta}$ is a diagonal matrix with diagonal elements $\eta_{00} = 1, \eta_{11} = \eta_{22} = \eta_{33} = -1$, space-time 4-vector $x = x^{\mu} = (ct,\ \boldsymbol{x})$ and $x_{\mu} = (ct, -\boldsymbol{x})$, 4-velocity $\frac{dx^{\alpha}}{d\tau} = \dot{x}^{\alpha} = (c\gamma_u,\ \boldsymbol{u}\gamma_u)$ is the 4-velocity with $\gamma_u = (1 - u^2/c^2)^{-\frac{1}{2}}$, and τ is the proper time along the particle's world-line, energy momentum four vector $p^{\alpha} = (p_0,\ \boldsymbol{p}) = \left(\frac{E}{c},\ \boldsymbol{p}\right)$, $\partial_{\alpha} \equiv \left(\frac{\partial}{c\partial t},\ \boldsymbol{\nabla}\right)$, $\partial^{\alpha} \equiv \left(\frac{\partial}{c\partial t}, -\boldsymbol{\nabla}\right)$, the *D'Alembertian* operator is $\partial_{\alpha}\partial^{\alpha} = \frac{\partial^2}{c^2\partial t^2} - \boldsymbol{\nabla}^2$, where Einstein's convention of sum over repeated indices is used. Newton's universal gravitational constant is *G*.

2. HMG from Local U(1) Phase Invariance

The free Dirac Lagrangian density of a massive Dirac particle of rest mass m_0 is given by

$$\mathcal{L} = i\hbar c\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - m_0 c^2\overline{\psi}\psi, \tag{1}$$

which is invariant under the transformation

$$\psi \rightarrow e^{i\theta}\psi \qquad \text{(global phase transformation)} \tag{2}$$

where θ is any real number. This is because under global phase transformation (2) $\overline{\psi} \rightarrow e^{-i\theta}\,\overline{\psi}$, which leaves $\overline{\psi}\psi$ in (1) unchanged as the exponential factors cancel out. But the Lagrangian density (1) is not invariant under the following transformation

$$\psi \rightarrow e^{i\theta(x)}\psi \qquad \text{(U(1) local phase transformation)} \tag{3}$$

where θ is now a function of space-time $x = x^{\mu} = (ct, \boldsymbol{x})$, because the factor $\partial_{\mu}\psi$ in (1) now picks up an extra term from the derivative of $\theta(x)$:

$$\partial_{\mu}\psi \rightarrow \partial_{\mu}\left(e^{i\theta(x)}\psi\right) = i\left(\partial_{\mu}\theta\right)e^{i\theta(x)}\psi + e^{i\theta(x)}\partial_{\mu}\psi \tag{4}$$

so that under local phase transformation,

$$\mathcal{L} \rightarrow \mathcal{L} - \hbar c\left(\partial_{\mu}\theta\right)\overline{\psi}\gamma^{\mu}\psi = \mathcal{L} - \left(\partial_{\mu}\frac{\hbar}{m_0}\theta\right)m_0 c\overline{\psi}\gamma^{\mu}\psi \tag{5}$$

where $m_0 \neq 0$ is the invariant mass of the particle. Now we introduce a new variable $\lambda(x)$:

$$\lambda(x) = -\frac{\hbar}{m_0}\theta(x). \tag{6}$$

In terms of λ, then,

$$\mathcal{L} \rightarrow \mathcal{L} + (m_0 c\,\overline{\psi}\gamma^{\mu}\psi)\partial_{\mu}\lambda \tag{7}$$

under the local transformation,

$$\psi \rightarrow e^{-\frac{im_0\lambda(x)}{\hbar}}\psi. \tag{8}$$

Now, *we demand that the complete Lagrangian be invariant under* U(1) *local phase transformations*. Since, the free Dirac Lagrangian (1) is not locally phase invariant, we are forced to add something to swallow up or nullify the extra term in equation (7). Specifically, we suppose

$$\mathcal{L} = \left[i\hbar c\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - m_0 c^2\overline{\psi}\psi\right] - (m_0 c\overline{\psi}\gamma^{\mu}\psi)A_{g\mu}, \tag{9}$$

where $A_{g\mu}$ is some new field, which changes in coordination with the local phase transformation according to the rule

$$A_{g\mu} \rightarrow A_{g\mu} + \partial_{\mu}\lambda. \tag{10}$$

This 'new, improved' Lagrangian is now locally invariant. But this was ensured at the cost of introducing a new vector field that couples to through the last term in equation (9). But the equation (9) is devoid of a `free' term for the field $A_{g\mu}$ itself. Since it is a vector, we look to the Proca-type Lagrangian [17]:

$$\mathcal{L}_{free} = \frac{\zeta}{4} f_{\mu\nu}f^{\mu\nu} + \zeta_0\left(\frac{\gamma_g c}{\hbar}\right)^2 A_g^{\mu}A_{g\mu} \tag{11}$$

where ζ and ζ_0 are some dimensional constants and γ_g is the mass of the free field $A_{g\mu}$. But there is a problem here, for whereas

$$f^{\mu\nu} = \left(\partial^{\mu}A_g^{\nu} - \partial^{\nu}A_g^{\mu}\right) \quad or \quad f_{\mu\nu} = \left(\partial_{\mu}A_{g\nu} - \partial_{\nu}A_{g\mu}\right) \tag{12}$$

is invariant under (10), $A_g^{\mu}A_{g\mu}$ is not. Evidently, the new field must be mass-less ($\gamma_g = 0$), otherwise the invariance will be lost. The complete Lagrangian then becomes

$$\mathcal{L} = \left[i\hbar c\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - m_0c^2\overline{\psi}\psi\right] + \mathcal{L}_{new}, \quad (13)$$

where $\mathcal{L}_{new} = \frac{\zeta}{4} f_{\mu\nu}f^{\mu\nu} - (m_0c\overline{\psi}\gamma^{\mu}\psi)A_{g\mu}$. (14)

Now by introducing a mass current density defined by

$j^{\mu} = m_0c(\overline{\psi}\gamma^{\mu}\psi)$, we rewrite (14) as (15)

$$\mathcal{L}_{new} = \frac{\zeta}{4} f_{\mu\nu}f^{\mu\nu} - j^{\mu}A_{g\mu}. \quad (16)$$

The equation of motion of this new field can be obtained using the Euler-Lagrange equations of motion:

$$\partial^{\beta}\frac{\partial\mathcal{L}_{new}}{\partial(\partial^{\beta}A_g^{\alpha})} = \frac{\partial\mathcal{L}_{new}}{\partial A_g^{\alpha}}. \quad (17)$$

A bit calculation (see for example, Jackson [17]) gives us

$$\frac{\partial\mathcal{L}_{new}}{\partial(\partial^{\beta}A_g^{\alpha})} = -\zeta f_{\alpha\beta} \quad \text{and} \quad \frac{\partial\mathcal{L}_{new}}{\partial A_g^{\alpha}} = -j_{\alpha}. \quad (18)$$

Using (18) in Euler-Lagrange eqs. (17), we get the equations of motion of the new field as

$$\partial^{\beta}f_{\alpha\beta} = \frac{1}{\zeta}j_{\alpha}. \quad (19)$$

Equation (19) expresses the generation of $f_{\alpha\beta}$ fields by the 4-current density of proper mass of neutral massive Dirac particles. However, for classical fields, the 4-current density of proper mass is represented by

$$j^{\alpha} = (\rho_0c,\ \boldsymbol{j}), \quad j_{\alpha} = (\rho_0c,\ -\boldsymbol{j}) \quad (20)$$

where $\boldsymbol{j} = \rho_0\boldsymbol{v}$, with ρ_0 = proper mass density and $\boldsymbol{v}$ = velocity of ρ_0. For static mass distributions, the current density $j_{\alpha} = j_0 = \rho_0c$. It produces a time-independent – static field, given by eq. (19):

$\frac{1}{c}\frac{\partial f_{00}}{\partial t} - \frac{\partial f_{01}}{\partial x} - \frac{\partial f_{02}}{\partial y} - \frac{\partial f_{03}}{\partial z} = \frac{\rho_0c}{\zeta}$, or since $f_{00} = 0$, we rewrite this equation as

$$\frac{\partial(cf_{01})}{\partial x} + \frac{\partial(cf_{02})}{\partial y} + \frac{\partial(cf_{03})}{\partial z} = -\frac{\rho_0c^2}{\zeta} \quad (21)$$

Equation (21) gives us Newton's gravitational field ($\boldsymbol{g}$) equation as expressed in Gauss's law of gravitostatics, viz.,

$$\boldsymbol{\nabla}\cdot\boldsymbol{g} = \frac{\partial g_x}{\partial x} + \frac{\partial g_y}{\partial y} + \frac{\partial g_z}{\partial z} = -4\pi G\rho_0 = -\frac{\rho_0}{\varepsilon_{og}} \quad \left(\text{by defining } \varepsilon_{0g} = \frac{1}{4\pi G}\right) \quad (22)$$

if we make the following identification:

$$f_{01} = g_x/c, \quad f_{02} = g_y/c,\ f_{03} = g_z/c, \quad \text{and} \quad \zeta = \frac{c^2}{4\pi G}. \quad (23)$$

With this value of ζ fixed by Newton's law of gravitation, not by us, eqs. (16) and (19) become

$$\mathcal{L}_{new} = \frac{c^2}{16\pi G} f_{\mu\nu}f^{\mu\nu} - j^{\mu}A_{g\mu} = \frac{c^2\varepsilon_{0g}}{4} f_{\mu\nu}f^{\mu\nu} - j^{\mu}A_{g\mu} \tag{24}$$

$$\partial^{\beta} f_{\alpha\beta} = \frac{4\pi G}{c^2} j_{\alpha} = \mu_{0g} j_{\alpha} \qquad \left(\text{Note that } \mu_{0g} = \frac{4\pi G}{c^2} \Rightarrow c = \frac{1}{\sqrt{\varepsilon_{0g}\mu_{0g}}}.\right) \tag{25}$$

which is applicable to Dirac current density (15) as well as classical current density (20). From the anti-symmetry property of $f^{\alpha\beta}$ $\left(f^{\alpha\beta} = -f^{\beta\alpha}\right)$, it follows form the results (23) that

$$f_{10} = -g_x/c, \quad f_{20} = -g_y/c, \; f_{30} = -g_z/c \text{ and } f_{\alpha\alpha} = 0. \tag{26}$$

The other elements of $f_{\alpha\beta}$ can be obtained as follows. For $\alpha = 1$, i.e., $j_1 = -j_x$, eq. (25) gives us

$$-\frac{4\pi G}{c^2} j_x = \frac{4\pi G}{c^2} j_1 = \partial^0 f_{10} + \partial^1 f_{11} + \partial^2 f_{12} + \partial^3 f_{13} = -\frac{1}{c^2}\frac{\partial g_x}{\partial t} - \frac{\partial f_{12}}{\partial y} - \frac{\partial f_{13}}{\partial z},$$

or

$$-\frac{4\pi G}{c^2} j_x = \begin{cases} -\frac{1}{c^2}\frac{\partial g_x}{\partial t} + (\boldsymbol{\nabla} \times \boldsymbol{b})_x & \text{(For MG)} \\ -\frac{1}{c^2}\frac{\partial g_x}{\partial t} - (\boldsymbol{\nabla} \times \boldsymbol{b})_x & \text{(For HG)} \end{cases} \tag{27}$$

where $f_{12} = -b_z$ and $f_{13} = b_y$ for Maxwellian Gravity (MG); $f_{12} = b_z$ and $f_{13} = -b_y$ for Heaviside Gravity (HG). This way, we determined all the elements of the anti-symmetric 'field strength tensor' $f_{\alpha\beta}$:

$$f_{\alpha\beta} = \begin{cases} \begin{pmatrix} 0 & g_x/c & g_y/c & g_z/c \\ -g_x/c & 0 & -b_z & b_y \\ -g_y/c & b_z & 0 & -b_x \\ -g_z/c & -b_y & b_x & 0 \end{pmatrix} & \text{(For MG)} \\ \begin{pmatrix} 0 & g_x/c & g_y/c & g_z/c \\ -g_x/c & 0 & b_z & -b_y \\ -g_y/c & -b_z & 0 & b_x \\ -g_z/c & b_y & -b_x & 0 \end{pmatrix} & \text{(For HG)} \end{cases} \tag{28}$$

and the Gravito-Ampère-Maxwell law of MG and HG:

$$\boldsymbol{\nabla} \times \boldsymbol{b} = \begin{cases} -\frac{4\pi G}{c^2}\boldsymbol{j} + \frac{1}{c^2}\frac{\partial \boldsymbol{g}}{\partial t} & \text{(For MG)} \\ +\frac{4\pi G}{c^2}\boldsymbol{j} - \frac{1}{c^2}\frac{\partial \boldsymbol{g}}{\partial t} & \text{(For HG)} \end{cases} \tag{29}$$

where $\boldsymbol{b}$ is named as gravitomagnetic field, which is generated by invariant mass current and time-varying gravitational or gravitoelectric field $\boldsymbol{g}$.

For reference, we note the field strength tensor with two contravariant indices:

$$f^{\alpha\beta} = \eta^{\alpha\gamma} f_{\gamma\delta} \eta^{\delta\beta} = \begin{cases} \begin{pmatrix} 0 & -g_x/c & -g_y/c & -g_z/c \\ g_x/c & 0 & -b_z & b_y \\ g_y/c & b_z & 0 & -b_x \\ g_z/c & -b_y & b_x & 0 \end{pmatrix} & \text{(For MG)} \\ \begin{pmatrix} 0 & -g_x/c & -g_y/c & -g_z/c \\ g_x/c & 0 & b_z & -b_y \\ g_y/c & -b_z & 0 & b_x \\ g_z/c & b_y & -b_x & 0 \end{pmatrix} & \text{(For HG)} \end{cases} \tag{30}$$

From eq. (25) and the anti-symmetry property of $f^{\alpha\beta}$, it follows that j^{α} is divergence-less:

$$\partial_\alpha j^\alpha = 0 = \boldsymbol{\nabla} \cdot \boldsymbol{j} + \frac{\partial \rho_0}{\partial t}. \tag{31}$$

This is the *continuity equation* expressing the local conservation of proper mass.

Equation (25) gives us two in-homogeneous equations of MG and HG. The very definition of $f_{\alpha\beta}$ in eq. (12), automatically guarantees us the Bianchi identity:

$$\partial_\alpha f_{\beta\gamma} + \partial_\beta f_{\gamma\delta} + \partial_\gamma f_{\alpha\beta} = 0, \tag{32}$$

(where α, β and γ are any three of the integers 0, 1, 2, 3), from which two homogeneous equations emerge naturally:

$$\boldsymbol{\nabla} \cdot \boldsymbol{b} = 0 \qquad \text{(For both MG and HG)} \tag{33}$$

$$\boldsymbol{\nabla} \times \boldsymbol{g} = \begin{cases} -\dfrac{\partial \boldsymbol{b}}{\partial t} & \text{(For MG)} \\ +\dfrac{\partial \boldsymbol{b}}{\partial t} & \text{(For HG)} \end{cases} \tag{34}$$

The Bianchi identity may concisely be expressed by the zero divergence of a dual field-strength tensor, $Ƒ^{\alpha\beta}$, viz.,

$$\partial_\alpha Ƒ^{\alpha\beta} = 0, \text{ where} \tag{35}$$

$$\mathrm{F}^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\beta\gamma\delta} f_{\gamma\delta} = \begin{pmatrix} 0 & -b_x & -b_y & -b_z \\ b_x & 0 & g_z/c & -g_y/c \\ b_y & -g_z/c & 0 & g_x/c \\ b_z & g_y/c & -g_x/c & 0 \end{pmatrix} \text{(For MG)} \quad (36)$$

and the totally anti-symmetric fourth rank tensor $\epsilon^{\alpha\beta\gamma\delta}$ (known as Levi-Civita Tensor) is defined by

$$\epsilon^{\alpha\beta\gamma\delta} = \begin{cases} +1 & \text{for } \alpha = 0, \beta = 1, \gamma = 2, \delta = 3, \text{and any even permutation} \\ -1 & \text{for any odd permutation} \\ 0 & \text{if any two indices are equal.} \end{cases} \quad (37)$$

The dual field-strength tensor $\mathrm{F}^{\alpha\beta}$ for HG can be obtained from eq. (36) by substitution $\boldsymbol{b} \to -\boldsymbol{b}$, with $\boldsymbol{g}$ remaining the same.

Equation (33) suggests that $\boldsymbol{b}$ can be defined as the curl of a vector function $\boldsymbol{A}_g$ (say). By defining

$$\boldsymbol{b} = \begin{cases} +\nabla \times \boldsymbol{A}_g & \text{(For MG)} \\ -\nabla \times \boldsymbol{A}_g & \text{(For HG)} \end{cases} \quad (38)$$

then using this equation in (34), we find

$$\boldsymbol{\nabla} \times \left(\boldsymbol{g} + \frac{\partial \boldsymbol{A}_g}{\partial t}\right) = \boldsymbol{0} \qquad \text{(For both MG and HG)} \quad (39)$$

which is equivalent to say that the vector quantity inside the parentheses of eq. (39) can be written as the gradient of a scalar potential, $A_{g0} = \frac{\varphi_g}{c}$:

$$\boldsymbol{g} = -\boldsymbol{\nabla}\varphi_g - \frac{\partial \boldsymbol{A}_g}{\partial t} \qquad \text{(For both MG and HG)} \quad (40)$$

In relativistic notation, equations (38) and (40) become

$$f^{\alpha\beta} = \partial^\alpha A_g^\beta - \partial^\beta A_g^\alpha, \quad (41)$$

(as they must because of their common origin) where

$$A_g^\alpha = \left(A_g^0, \boldsymbol{A}_g\right) = (\varphi_g/c\,, \boldsymbol{A}_g) \ \text{ and } \ A_{g\alpha} = \left(A_{g0}, -\boldsymbol{A}_g\right) = (\varphi_g/c\,, -\boldsymbol{A}_g) \quad (42)$$

In terms of $A_{g\alpha}$, the in-homogeneous equations (25) of MG and HG read:

$$\partial^\beta \partial_\beta A_{g\alpha} - \partial_\alpha\left(\partial^\beta A_{g\beta}\right) = -\frac{4\pi G}{c^2} j_\alpha \quad (43)$$

Under the gravito-Lorentz condition: $\partial^\beta A_{g\beta} = 0$, (44)

the in-homogeneous equation (43) simplifies to the following equation:

$$\partial^{\beta}\partial_{\beta}A_{g\alpha} = \left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)A_{g\alpha} = -\frac{4\pi G}{c^2}j_{\alpha} = -\mu_{0g}j_{\alpha} \quad \text{(For MG and HG).} \quad (45)$$

In electromagnetism, the equation corresponding to equation (45) is

$$\partial^{\beta}\partial_{\beta}A_{e\alpha} = \left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)A_{e\alpha} = \square\square\mu_0 j_{e\alpha} \quad (46)$$

where the electromagnetic 4-vector potential $A_{e\alpha}$ and the 4-current $j_{e\alpha}$ are

$$A_{e\alpha} = \left(\frac{\varphi_e}{c}, -\boldsymbol{A}_e\right) \text{ and } j_{e\alpha} = (\rho_e c, -\boldsymbol{j}_e). \quad (47)$$

with the symbols having their usual meanings. The crucial sign difference between the equations (45) and (46) will explain why two like static masses attract each other, while two like static charges repel each other as we shall see. Since the fundamental field equations are the same for MG and HG, they represent the same physical thing and any sign difference in some particular terms arises due to particular definitions which will not change the physics. Hence, in what follows, what we call MG is to be understood as HMG.

As per the present formulation of MG, the relativistic Lagrangian (not Lagrangian density) for a single particle of rest mass m moving in the external field of MG, is written as

$$L_{MG} = -\left[m_0 c\sqrt{\eta^{\alpha\beta}\frac{dx_\alpha}{ds}\frac{dx_\beta}{ds}} + m_0 c\frac{dx_\alpha}{ds}A_g^{\alpha}(x)\right] \quad (48)$$

where $ds = cd\tau$; τ is the proper time along the particle's world-line. From this Lagrangian, one obtains the co-variant equation of motion following the usual procedure of classical electrodynamics:

$$\frac{d^2x^{\alpha}}{d\tau^2} = f^{\alpha\beta}\frac{dx_\beta}{d\tau} \quad (49)$$

Equation (49) shows us that the proper acceleration of a particle in the fields of MG is independent of its rest mass m_0. Equation (49) is the relativistic generalization of Galileo’s law of Universality of Free Fall (UFF) - known to be true both theoretically and experimentally since Galileo’s time. It states that all particles of whatever rest masses moving with same proper velocity $dx_\beta/d\tau$ in a given gravitational field $f^{\alpha\beta}$ experience the same proper acceleration. It is to be noted that the equation of motion (49) holds only in an inertial frame. Appropriate modifications are necessary for its application in non-inertial frames, as is done in non-relativistic physics.

Now, if we introduce the energy momentum four vector:

$$p^{\alpha} = (p^0, \boldsymbol{p}) = m_0(U^0, \boldsymbol{U}) \text{ where } p^0 = p_0 = \frac{E}{c} \text{ and } U^{\alpha} = (\gamma c,\ \gamma \boldsymbol{v}), \tag{50}$$

then we can re-write Eq. (49) in terms of p_{β} as

$$\frac{dp^{\alpha}}{d\tau} = f^{\alpha\beta} p_{\beta}. \tag{51}$$

Thus, the energy-momentum 4-vector of all particles couples to the fields of MG.

3. Attraction Between Static Masses

The static interaction between two point (positive) masses at rest can be obtained following a classical approach [18] within the framework of Maxwellian Gravity as shown in ref. [19] where we have considered a charged Dirac particle instead of a neutral Dirac particle considered here. However to find the Newtonian attraction between two static masses, here we adopt Feynman's field theoretical approach [2] as follows.

The source of gravito-electromagnetism is the vector current j_{μ}, which is related to vector potential $A_{g\mu}$ by the relation

$$A_{g\mu} = \frac{4\pi G}{c^2} \frac{1}{k^2} j_{\mu} = \mu_{0g} \frac{1}{k^2} j_{\mu} \quad \text{(In electromagnetism: } A_{e\mu} = -\mu_0 \frac{1}{k^2} j_{e\mu} \text{)} \tag{52}$$

Here we have taken Fourier transforms and used the momentum-space representation. The D'Alembertian operator $\partial^{\alpha}\partial_{\alpha}$ in eq. (45) is simply $-k^2$ in momentum-space. As in electromagnetism the calculation of amplitudes in gravito-electromagnetism is made with the help of propagators connecting currents in the manner as symbolized by Feynman diagrams as that in Figure 1. The amplitudes for such processes are generally computed as a function of relativistic invariants restricting the answer as demanded by rules of momentum and energy conservation. As in electromagnetism, the guts of gravito-electromagnetism are contained in the specification of the interaction between a mass current and the field as $j^{\mu} A_{g\mu}$; in terms of the sources, this becomes an interaction between two currents:

$$j'_{\mu} A^{\mu}_{g} = \frac{4\pi G}{c^2} j'_{\mu} \frac{1}{k^2} j^{\mu} \tag{53}$$

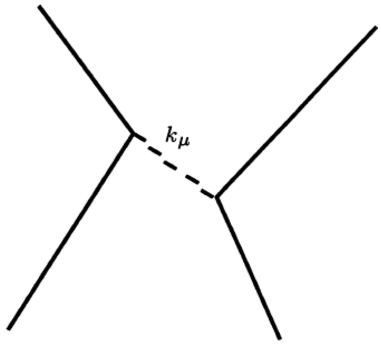

Fig. 1. Feynman Diagram

In our choice of coordinates and units, $k^\mu = \left(\frac{\omega}{c},\ 0,\ 0,\ \kappa\right)$, $k_\mu = \left(\frac{\omega}{c},\ 0,\ 0,\ -\kappa\right)$ and $A_{g\mu}$ is given by eq. (52). Then the current-current interaction when the exchanged particle has a momentum k^μ is given by

$$\frac{4\pi G}{c^2} j'_\mu \frac{1}{k^2} j^\mu = \frac{4\pi G}{c^2} \frac{1}{\frac{\omega^2}{c^2} - \kappa^2} \left(j'_0 j_0 - j'_1 j_1 - j'_2 j_2 - j'_3 j_3\right) \quad \text{or} \tag{54}$$

$$\frac{4\pi G}{c^2} j'_\mu \frac{1}{k^2} j^\mu = \frac{4\pi G}{\omega^2 - c^2\kappa^2} \left(j'_0 j_0 - j'_1 j_1 - j'_2 j_2 - j'_3 j_3\right) \tag{55}$$

The conservation of proper mass, which states that the four divergence of proper mass current is zero, in momentum-space becomes simply the restriction

$$j_\mu k^\mu = 0. \tag{56}$$

In the coordinate system we have chosen, this restriction connects the third and the zeroth component of the currents by

$$\frac{\omega}{c} j_0 - \kappa j_3 = 0 \quad or \quad j_3 = \frac{\omega}{\kappa c} j_0 \tag{57}$$

If we insert this expression for j_3 into the amplitude in eq. (55), we get

$$\frac{4\pi G}{c^2} j'_\mu \frac{1}{k^2} j^\mu = -\frac{4\pi G}{c^2\kappa^2} j'_0 j_0 - \frac{4\pi G}{\omega^2 - c^2\kappa^2} \left(j'_1 j_1 + j'_2 j_2\right) \tag{58}$$

Now we can give interpretation to the two terms in eq. (58). The zeroth component of the current is simply the mass density; in the situation where we have stationary masses, it is the only non-zero component of current. The first term is independent of frequency; when we take the inverse Fourier transform to convert this to a space-interaction, we find that it represents an instantaneously acting Newton potential.

$$(F.T.)^{-1}\left(-\frac{4\pi G}{c^2\kappa^2} j'_0 j_0\right) = -\frac{G m'_0 m_0}{r}\,\delta(t - t'). \tag{59}$$

This is always the leading term in the limit of small velocities. The term appears instantaneous, but this is only due to the separation we have made into two terms is not manifestly co-variant. The total interaction is really an invariant quantity; the second term represents corrections to the instantaneous Newtonian interaction. The force in eq. (59) is attractive, if m_0' and m_0 are of the same sign and repulsive if they are of the opposite sign - the reverse case of electrical interaction between two static electric charges. Instead of Feynman's [2] approach adopted above one can adopt Zee's [5] path-integral approach to get at the same conclusion if one uses our equations (45) and (24).

4. Lagrangian For Quantum Gravitodynamics

According to the present study, the final expression for the Lagrangian for quantum gravitodynamics (QGD) of neutral massive Dirac fields interacting with fields of Maxwellian Gravity (spin-1 gravitons) in flat space-time turns out as

$$\mathcal{L}_{QGD} = \left[i\hbar c\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - m_0 c^2\overline{\psi}\psi\right] + \frac{c^2}{16\pi G} f_{\mu\nu}f^{\mu\nu} - j^{\mu}A_{g\mu} \qquad (60)$$

This Lagrangian may be used to study the gravitational interaction of electrically neutral massive spin half particles such as the neutrons and massive neutrinos.

5. Remarks on 'Misner, Thorne and Wheeler on HMG'

Misner, Thorne and Wheeler [20], in their 'Exercises on flat space-time theories of gravity', have considered a possible vector theory of gravity described by the Lagrangian density in eq. (24) within the framework of special relativity. They found it to be deficient in that there is no bending of light, perihelion advance of Mercury and negative energy gravitational waves in vector theory. However, our preliminary results from two different considerations in ref. [19] show that gravitational waves emanating from self-gravitating systems always carry positive energy and momentum in spite of the fact that the intrinsic field energy of static gravitoelectromagnetic fields is negative. As regards the classical tests of General Relativity, there are reports [21, 22, 23, 24, 25] of their explanation in the framework of vector theory. Hilborn [26] recently discussed gravitational waves from orbiting binaries without general relativity but using vector gravity and his results in certain terms agree with those of general relativity and differ from GR in some other terms. The speed of gravitational waves in vacuum $c_g = c$ in HMG which is in agreement with recent experimental data [27]. However there is no unique and unambiguous value of c_g in different versions of linearized gravity as explicitly discussed recently in refs. [9, 19].

6. Emergence of MG from other Approaches

There exist several classical approaches to get the fundamental equations of MG, which include (a) Galileo-Newtonian Relativistic approach [9, 10, 28], (b) use of principle of causality [16, 29], (c) axiomatic approaches common to electromagnetism and gravito-electromagnetism [30, 31], (d) special relativistic approach [9, 32, 33] and (d) a specific lilearized version of General Relativity (GR) [34]. Thus our findings of the basic equations MG here corroborate all of these previous findings.

7. Conclusions

Using the principle of local phase (or gauge) invariance of the free Lagrangian density of a neutral massive Dirac particle and Newton's law of gravity, we rediscovered the spin-1 Heaviside-Maxwellian Gravity, where like masses are shown to attract each other under static conditions, contrary to the standard view of field theorists. The theory looks interesting and important, particularly in respect of its quantization and unification with other fundamental forces of nature. It may shed some new light on our understanding of the nature of physical interactions and their interplay at the quantum level.

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